

# License Auctions when Winning Bids are Financed through Debt

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SOM Research Report 06F06

## Abstract

We study an auction where two licenses to operate on a new market are sold, and winning bidders finance their bids on the debt market. Higher bids imply higher debts, which affects product market competition. We compare our results to those of a beauty contest and a standard auction. For the case that debt induces firms to compete more aggressively, we find that consumer prices are lower, and expected firm profits are strictly positive although firms are a priori identical. When debt induces firms to compete less aggressively, we find that firms make zero profits, and consumer prices are higher.

JEL Classification Codes: D44, D45, L13

Keywords: License Auctions, Debt, Oligopoly

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# 1 Introduction

Over the last decade, license auctions in the US and Europe sparked a huge interest from both academics and the general public. In the US, the FCC auctioned licenses to use the electromagnetic spectrum for personal communication services. Between July 1994 and July 1998, 16 auctions were held, where 5,893 licenses were sold. Total revenues amounted to \$22.9 billion dollars.<sup>1</sup> Throughout Europe, licenses for "third generation" (3G) mobile telecommunication (or UMTS) took place during 2000 and 2001.<sup>2</sup> These auctions, held in 9 countries, raised over \$100 billion, or over 1.5% of GDP. The revenue per inhabitant differed greatly per country.<sup>3</sup> Currently, European countries prepare to auction off the 3G expansion band.<sup>4</sup>

Traditional auction theory<sup>5</sup> may not be the most appropriate framework to study these auctions. Indeed, Klemperer (2002b) argues that in analyses of license auctions based on this literature, often too much attention is given to technicalities concerning asymmetric information, and too little attention to market structure and industrial organization aspects. Traditional models typically assume that for each bidder the value of the object that is being auctioned is fixed and given. In a license auction, this is often not the case. Here, firms bid on the right to compete on a market. The willingness to pay for that right will depend on the characteristics of the aftermarket (see e.g. Jehiel and Moldovanu, 2003). For example, a firm is willing to pay a higher amount to compete against an inefficient firm than it is to compete against an efficient firm, simply because its profits will be higher in the former case. Bidding behavior in a license auction can also be affected by the market structure that exists *before* the auction. Incumbents may be inclined to pre-empt new entry in the market. The ability to do so will depend on the auction format (Hoppe et

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<sup>1</sup>Cramton and Schwartz (2000); for more on the design of these auctions, see e.g. McAfee and McMillan (1996), or McMillan (1994).

<sup>2</sup>See Van Damme (2002), or Börgers and Dustmann (2003).

<sup>3</sup>Klemperer (2002a).

<sup>4</sup>See The Economist (2005).

<sup>5</sup>For a survey see Wolfstetter (1996), or Klemperer (1999).

al., 2006). Also, the bid that a firm submits in an auction may serve as a signal for some private information that is relevant for the aftermarket (Goeree, 2003).

This paper studies another important characteristic of the post-auction interaction between firms that affects their bidding behavior in a license auction. One striking aspect of license auctions is that winning firms often have to take on debt to be able to finance their bid (see e.g. *The Economist*, 2002). From the industrial organization literature, it is well-known that the competitive behavior of a firm is affected by the amount of debt that it holds. In a seminal article, Brander and Lewis (1986) study a model in which firms hold debt and compete in Cournot fashion. Firms face uncertainty with respect to e.g. the level of marginal costs. After output decisions have been made, uncertainty is resolved. Each firm has limited liability. If operating profits fall short of its debt level, the firm is simply not able to repay its debt, and hence has a zero net profit. This will be the case if marginal costs turn out to be high. A firm that maximizes net profits will thus focus on cases with low marginal costs when making its output decision. This implies that it will choose a higher output than it would without limited liability. In other words, in this context, more debt induces firms to compete more aggressively. Yet, there may also be cases in which more debt induces firms to compete less aggressively (Showalter, 1995).

In license auctions, the bid of a firm will depend on the profits it expects to make in the aftermarket. But that profit will depend on the debt levels that the winners of the auction take on. In turn, these debt levels depend on the bids in the auction stage. In this paper, we model this issue. Two licenses to operate on some new market are being auctioned. The two winners of the auction will establish a duopoly, and finance their bids on a competitive debt market. A number of a priori identical firms participate in the auction. After the auction, and after financing the bids, the two winners compete on the market. We assume that firms have symmetric but incomplete information concerning some aspect of the aftermarket. When firms place higher bids, they also have to take on higher debts, which in turn affects their behavior on the product market. Firms take all

these effects into account when placing their bids. We study how bidding behavior on the auction market is ultimately affected, and the prices and profits that will ultimately result.

Our model yields some surprising insights. Consider the case in which debt induces firms to compete more aggressively. We find that the winners of the auction then have expected profits that are strictly positive. This is surprising since a priori all firms are identical and our model does not have asymmetric information. Hence, one would expect profits to be driven down to zero. This is not the case. Effectively, the equilibrium of our model exhibits credit rationing. Winning bidders, even though they make positive expected profits, will not be outbid. Any higher bid would yield a debt level that implies negative net expected profits for debtholders. A higher level of debt induces firms to compete more aggressively on the product market, reducing the probability that the debt will be repaid. Therefore, the supply curve of debt is backward bending. Thus, a firm would be willing to bid more if it could obtain funding, but it cannot.

With respect to license auctions, one of the main concerns in the popular press is that their use will increase consumer prices. Firms will recoup the costs of licenses by simply adding a mark-up to consumer prices, the argument goes. Of course, this argument is invalid, as prices paid at an auction are simply sunk costs. In our model, higher fees even lead to *lower* prices. This is due to the strategic effect of debt: as firms take on more debt, they will compete more aggressively on the product market.<sup>6</sup> Thus, consumer prices in our model are lower than in a beauty contest, or in a situation where winners can finance their bids out of internal funds and do not have to resort to the debt market to finance their bids.

Yet, as already mentioned, there are cases in which debt induces firms to compete less aggressively, e.g. in a world with price setting and uncertainty about demand (see Showalter 1995). In that case, debts cannot be repaid if demand turns out to be low. In setting their price, firms thus focus on cases with high demand, which implies that they

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<sup>6</sup>We also made this argument, but only intuitively, in Haan and Toolsema (2000).

set higher prices than they would do in a case without debt. Such a framework has the following implications for our model. First, winners of the auction have zero expected profits, as there is no credit rationing. Second, equilibrium consumer prices are higher than they are in a case where firms obtain their license for free. Third, winning bids are higher with debt than they are when firms have internal funds.

A few papers study other aspects of the interplay between debt and auctions. Chowdhry and Nanda (1993) also study the strategic role of debt in an auction – but in a context entirely different from ours. They study a takeover contest, in which many raiders bid to take over a firm. Clayton and Ravid (2002) study the effect of the initial debt level of a firm on its bidding behavior in the US FCC auctions. They find that, as debt levels increase, firms tend to reduce their bids. Note however that they study the amount of debt a firm already has when the auction takes place. We study the amount of debt a firm has to take on as a consequence of winning the auction. Moreover, in our model, the amount of debt is endogenously determined. In Zheng’s (2001) model, bidders differ with respect to the amount of funds that they have. Firms with lower funds have to take on higher debts upon winning the auction. That implies that these bidders are willing to risk more, and therefore bid more aggressively. It also implies that the winners of the auction are exactly those bidders that are more likely to go bankrupt. In Zheng (2001), limited liability therefore has a direct effect on bidding behavior. In our model, the effect is indirect, and operates via the competition stage that takes place after the auction.

In our paper, we primarily focus on the first case sketched above, where debt induces firms to compete more aggressively. In the next section, we present that model. We do not specify the exact mode of competition. It suffices to put some weak assumptions on the equilibrium profits of the competition stage. In section 3 we solve the model. In section 4 we summarize our main results, and we give some numerical examples in section 5. These examples cover the cases of Hotelling, Bertrand, and Cournot competition. Section 6 considers the case in which debt induces firms to compete less aggressively. Section 7

concludes.

## 2 The model

We consider a three-stage model. In the first stage, the *auction stage*,  $N > 2$  bidders compete in a sealed-bid license auction, where winning bidders pay their own bid. Bidders are ex ante identical. The two highest bidders obtain a license to operate in a new market. Without loss of generality, the highest bidder will be denoted firm 1 and the second highest bidder will be denoted firm 2. Their bids are denoted  $b_1$  and  $b_2$ . In the case of ties, winners will be decided by coin toss. In stage 2, the *debt stage*, the two winning firms finance their bids on a perfectly competitive debt market. Firm  $i$  obtains an amount  $b_i$  to pay for its bid, against the promise of repaying  $d_i$  at the end of the game. We assume that if it is not the case that both firms are able to secure financing in the debt stage, then the auction will be declared void, and a new auction will be held. In stage 3, the *competition stage*, the two winning firms compete on the output market, where they face uncertainty. After firms have chosen their strategic actions, uncertainty is resolved, consumers make their purchase decisions and, if possible, debts are repaid. Without loss of generality, we assume no discounting.

The strategic effect of debt may work in two opposite directions. More debt may result in more aggressive competition, that is, higher quantities and lower prices. This is the case, for example, with Cournot competition and uncertainty about either marginal cost or demand (Brander and Lewis, 1986), and with Bertrand competition and uncertainty about marginal cost (Showalter, 1995). However, it may also be the case that more debt results in less aggressive competition, that is, in lower quantities and higher prices. This is the case, for example, with Bertrand competition and uncertainty about demand (Showalter, 1995). In this paper, we mainly focus on the former case. The latter case is solved in section 6. For simplicity, we assume in our model that firms can only take on debt to finance their bids in the license auction. Hence, we do not allow firms to take on debt

solely for strategic reasons, as they do in the literature referred to above.

In the remainder of this section, we give a detailed description of the three stages of the model: the auction stage, the debt stage, and the competition stage. We describe these stages in the same order as we solve the model: by backward induction, starting with the last stage. We then give a formal definition of the equilibrium concept that we use. We end with some technical assumptions.

**Competition stage** In this stage, the two firms that have won the auction in stage 1, and managed to obtain funding, compete on the output market. Products are substitutes. Demand functions are downward sloping. Firms compete by simultaneously choosing an action  $a$ , which may refer to either price or quantity. Let  $a_i \geq 0$  denote the action chosen by firm  $i$ . Operating profits of each firm will depend on the actions chosen by both firms, and the realization of some random variable  $\omega$ . Firm  $i$ 's operating profits are denoted  $\pi_i(a_i, a_j, \omega)$ , which is continuous and strictly concave in  $a_i$ .

The uncertainty reflected by  $\omega$  may concern e.g. the marginal costs firms will incur, or the level of demand that they will face. We use the convention that higher values of  $\omega$  correspond to less favorable states of the world in which, *ceteris paribus*, operating profits are lower:

$$\frac{\partial \pi_i(a_i, a_j, \omega)}{\partial \omega} < 0.$$

If the uncertainty concerns marginal costs, low  $\omega$  thus reflects low marginal costs. If the uncertainty concerns demand, low  $\omega$  reflects high demand. The realization of  $\omega$  is drawn from a continuously differentiable, strictly positive probability density function  $f(\omega)$  with domain  $[\underline{\omega}, \bar{\omega}]$ . As usual, we assume that  $\pi_i$  is strictly concave in  $a_i$ . For expositional convenience, we also assume that  $\pi_i(a_i, a_j, \bar{\omega}) \geq 0$  for all relevant  $a_i$  and  $a_j$ .

Firms face limited liability. Firm  $i$  can just repay its debt  $d_i$  if  $\omega$  is such that  $\pi_i(a_i, a_j, \omega) = d_i$ . In that case, operating profits are just sufficient to cover the promised repayment  $d_i$ . Denote the value of  $\omega$  for which this equality is satisfied as  $\hat{\omega}_i(a_i, a_j, d_i)$ . If  $\omega \leq \hat{\omega}_i(a_i, a_j, d_i)$ ,

firm  $i$  is able to fully repay its debt. Debtholders then receive  $d_i$ . Thus, with  $\omega \in (\hat{\omega}_i(a_i, a_j, d_i), \bar{\omega}]$ , the firm makes positive operating profits, but these are insufficient to repay the debt  $d_i$ . In that case, all operating profits will be paid to the debtholders, and the firm's net profits are zero.

We denote the expected net profits of firm  $i$ , i.e. after the repayment of its debt, as  $\Pi_i(a_i, a_j, d_i)$ . We thus have

$$\Pi_i(a_i, a_j, d_i) = \int_{\underline{\omega}}^{\hat{\omega}_i(a_i, a_j, d_i)} (\pi_i(a_i, a_j, \omega) - d_i) f(\omega) d\omega. \quad (1)$$

We look for a Nash equilibrium in the competition subgame, and assume existence, uniqueness and stability of that equilibrium.

**Debt stage** In the debt stage, the two firms that have won the auction have to find debtholders willing to lend them the money to pay for their bids. A winning firm has submitted a bid  $b_i \geq 0$ . A debt contract can be represented by  $(b_i, d_i)$ : firm  $i$  receives the amount  $b_i$  now, in return for the promise to repay  $d_i$  at the end of the game. Since we assume a perfectly competitive debt market, firms can make a take-it-or-leave-it offer to debtholders. For ease of exposition, we will write  $\hat{\omega}_i(d_i, d_j) \equiv \hat{\omega}_i(a_i^*(d_i, d_j), a_j^*(d_j, d_i), d_i)$ . Also, we define the continuation profits as  $\pi_i^*(d_i, d_j, \omega) \equiv \pi_i(a_i^*(d_i, d_j), a_j^*(d_j, d_i), \omega)$ . The expected repayment to debtholders  $R_i(d_i, d_j)$  can then be written as

$$R_i(d_i, d_j) = \Pr(\omega \leq \hat{\omega}_i(d_i, d_j))d_i + \int_{\hat{\omega}_i(d_i, d_j)}^{\bar{\omega}} \pi_i^*(d_i, d_j, \omega) f(\omega) d\omega. \quad (2)$$

Here, the subscript  $i$  denotes that these debtholders lend to firm  $i$ . The expected net profits to debtholders now equal  $R_i(d_i, d_j) - b_i$ . The expected net profits of firm  $i$  at this stage can be written as

$$\Pi_i(d_i, d_j) \equiv \Pi_i(a_i^*(d_i, d_j), a_j^*(d_j, d_i), d_i) = \int_{\underline{\omega}}^{\hat{\omega}_i(d_i, d_j)} (\pi_i^*(d_i, d_j, \omega) - d_i) f(\omega) d\omega. \quad (3)$$

Firms 1 and 2 simultaneously set  $d_1$  and  $d_2$  such that their expected profits are maximized, given what will occur in the competition stage. If it is not the case that both firms are



able to secure financing, then the auction will be declared void and a new auction will be held. Thus, a firm that is able to secure financing will also lose its license if its competitor is not able to do so. Without making this assumption, a firm could simply submit a bid in the auction that is so high that the debt market is only willing to finance that bid if the other firm does not obtain financing. A firm is then able to effectively shut any competitor out of the market and obtain a monopoly.<sup>7</sup>

**Auction stage** In the auction stage,  $N > 2$  identical firms submit a bid to obtain a license. Firm  $k$  submits  $B_k$ ,  $k \in \{1, \dots, N\}$ . Expected net profits of firm  $k$  at the auction stage can then be denoted  $\Pi_k^A(B_1, \dots, B_N)$ . The highest bid is  $b_1 \equiv \max(B_1, \dots, B_N)$ , the second highest bid  $b_2 \equiv \max(B_1, \dots, B_N) \setminus \{b_1\}$ . In case ties occur, we define  $T$  as the number of firms that have submitted the same bid as the second-highest bidder:  $T \equiv \#\{k | B_k = b_2\}$ . Note that the highest bidder may also be among these. Given the vector of bids, the probability of obtaining a license for firm  $k$  now equals

$$P_k(B_1, \dots, B_N) = \begin{cases} 0 & \text{if } B_k < b_2 \\ \frac{1}{T} & \text{if } B_k = b_2 < b_1 \\ \frac{2}{T} & \text{if } B_k = b_1 = b_2 \\ 1 & \text{if } B_k = b_1 > b_2 \end{cases}$$

and we have

$$\Pi_k^A(B_1, \dots, B_N) = P_k(B_1, \dots, B_N) \cdot \Pi_i(d_i^*(B_k, B_{-k}), d_j^*(B_{-k}, B_k)), \quad (4)$$

with  $B_{-k} = \max(B_1, \dots, B_N) \setminus \{B_k\}$ .

**Equilibrium concept** Putting together all the elements of the three subgames, we can now define the subgame perfect Nash equilibrium of the full game as follows:

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<sup>7</sup>To be really precise, we also have to make the assumption that if an auction is rescheduled, the original winners both incur additional costs  $\varepsilon$ , which can be infinitely small. Without this additional assumption, we have an infinite number of equilibria in which all firms always submit very high bids, and the auction is never resolved.

**Definition 1** A Subgame Perfect Nash Equilibrium of the game described above consists of bids  $(B_1^*, \dots, B_N^*)$  for all bidders, and debt levels  $(d_1^*, d_2^*)$  and actions  $(a_1^*, a_2^*)$  for the two highest bidders, such that we have:

1. Equilibrium at the competition stage:

$$a_i^* \in \arg \max_{a_i} \Pi_i(a_i, a_j^*, d_i), \quad (5)$$

for  $i = 1, 2$  and  $j \neq i$ , and  $\Pi_i(a_i, a_j, d_i)$  as defined in (1);

2. Equilibrium at the debt stage:

$$\begin{aligned} d_i^* &\in \arg \max_{d_i} \Pi_i(a_i^*(d_i, d_j^*), a_j^*(d_j^*, d_i), d_i), \\ \text{s.t. } &\begin{cases} R_i(d_i, d_j^*) \geq b_i, \\ R_j(d_j^*, d_i) \geq b_j, \end{cases} \end{aligned} \quad (6)$$

for  $i = 1, 2$  and  $j \neq i$ , and  $R_i(d_i, d_j)$  as defined in (2);

3. Equilibrium at the auction stage:

$$B_k^* \in \arg \max_{B_k} \Pi_k^A(B_1^*, \dots, B_{k-1}^*, B_k, B_{k+1}^*, \dots, B_N^*)$$

for all  $k \in \{1, \dots, N\}$ , with  $\Pi_k^A(B_1, \dots, B_N)$  as defined in (4).

**Further technicalities** There are cases in which a firm cannot possibly make strictly positive profits. As the amount of debt  $d_i$  that a firm holds increases, there is a point where it is impossible for firm  $i$  to make strictly positive net profits regardless of the action  $a_i$  that it chooses, even if we are in the most favorable state of the world, so if  $\omega = \underline{\omega}$ . Formally, for any  $d_j$  there is a  $\tilde{d}_i(d_j)$  such that  $\pi_i^*(\tilde{d}_i(d_j), d_j, \underline{\omega}) = \tilde{d}_i(d_j)$ . For firm  $i$  to be able to make strictly positive expected profits, we need  $d_i < \tilde{d}_i(d_j)$ . If this is not satisfied, then any action  $a_i$  will yield a zero profit. For technical convenience, we assume that for any  $d_i \geq \tilde{d}_i(d_j)$ , we have  $a_i^*(d_i, d_j) = \lim_{D \nearrow \tilde{d}_i(d_j)} a^*(D, d_j)$ . Thus, the firm will choose the same action that it would choose with the highest  $d_i$  that could still yield positive profits.

Consider an increase in the debt level of a firm's competitor. For our analysis, we require that this has an unambiguous effect on the firm's operating profits, independent of the realization of the random variable  $\omega$ . Thus:

**Assumption 1** *The derivative  $\frac{\partial \pi_i^*(d_i, d_j, \omega)}{\partial d_j}$  has the same sign for all  $\omega, d_i$ .*

Suppose that both firms have the same debt level  $d$ . We will refer to such a case as one with a common debt level. We make a similar assumption as the one above for a change in that common debt level:

**Assumption 2** *The derivative  $\frac{d\pi_i^*(d, d, \omega)}{dd}$  has the same sign for all  $\omega$ .*

For the bulk of this paper, we will consider the case in which more debt induces firms to compete more aggressively. We will show that the following assumption is sufficient for that to hold:

**Assumption 3** *Marginal profit is strictly decreasing in  $\omega$ . That is,  $\frac{\partial^2 \pi_i}{\partial \omega \partial q_i} < 0$ , with  $q_i$  the quantity sold by firm  $i$ .*

In section 6, we consider the opposite case, in which more debt makes firms compete less aggressively. In section 5, we show that the assumptions above are satisfied for the cases of Hotelling, Bertrand and Cournot competition with cost uncertainty and linear demand, and also for Cournot competition with demand uncertainty and linear demand.

## 3 Solving the model

### 3.1 Preliminaries

To derive the equilibrium of our model, we need some preliminary results, which we establish in a number of lemmas. All proofs are in the appendix.

First, with quantity competition, more debt induces a firm to compete more aggressively, in the sense that its best-reply function shifts upwards. With more debt, a given

output of the competitor will lead to a higher output for this firm. Also, with price competition, more debt induces a firm to compete more aggressively, but that now implies that its best-reply function shifts downwards. With more debt, a given price of the competitor will lead to a lower price for this firm (Lemma 1). As a result, we have that a firm's operating profit decreases if its competitor takes on higher debt (Lemma 2). Formally, denote the best-reply function of firm  $i$  as  $\beta_i(a_j; d_i)$ .

**Lemma 1** *Having more debt induces a firm to compete more aggressively:*

$$\frac{d\beta_i}{dd_i} \begin{cases} > 0 & \text{if } a_i \equiv q_i \text{ and } d_i \in [0, \tilde{d}_i(d_j)], \\ < 0 & \text{if } a_i \equiv p_i \text{ and } d_i \in [0, \tilde{d}_i(d_j)], \\ = 0 & \text{if } d_i > \tilde{d}_i(d_j). \end{cases}$$

**Lemma 2** *An increase in a firm's debt level decreases the competitor's equilibrium operating profits:*

$$\frac{\partial \pi_i^*(d_i, d_j, \omega)}{\partial d_j} \begin{cases} < 0 & \text{if } d_i \in [0, \tilde{d}_i(d_j)], \\ = 0 & \text{otherwise,} \end{cases} \quad \forall \omega \in [\underline{\omega}, \bar{\omega}].$$

It will also prove useful to consider the case in which both firms have the same level of debt  $d$ , and to study the effect of an increase in that common debt level. Similar to  $\tilde{d}_i(d_j)$ , we can define  $\tilde{d}$  as that value of the common debt level for which firms are not able to make positive profits, regardless of the value of  $\omega$ . Thus,  $\pi_i^*(\tilde{d}, \tilde{d}, \underline{\omega}) = \tilde{d}$ . Again, we assume that if  $d$  increases beyond  $\tilde{d}$  the behavior of the firms does not change. Thus for any  $d \geq \tilde{d}$ , we have  $a^*(d, d) = \lim_{D \nearrow \tilde{d}} a^*(D, D)$ . Define the expected operating profits of firm  $i$  as follows:

$$E_\omega(\pi_i^*(d_i, d_j)) \equiv \int_{\underline{\omega}}^{\bar{\omega}} \pi_i^*(d_i, d_j, \omega) f(\omega) d\omega.$$

Note that firms are a priori identical, so whenever  $d_1 = d_2 = d$ , we may drop the subscript  $i$ . We can now show:

**Lemma 3** *An increase in the common debt level decreases the expected operating profits of the firms:*

$$\frac{dE_{\omega}(\pi^*(d, d))}{dd} \begin{cases} < 0 & \text{if } d \in [0, \tilde{d}], \\ = 0 & \text{otherwise.} \end{cases}$$

Now consider the expected revenues for debtholders:

**Lemma 4** *An increase in the amount of debt that a firm holds, decreases the expected revenue for debtholders of the other firm:*

$$\frac{\partial R_i(d_i, d_j)}{\partial d_j} \begin{cases} < 0 & \text{if } d_i \in [0, \tilde{d}_i(d_j)], \\ = 0 & \text{otherwise.} \end{cases}$$

The intuition is as follows. From Lemma 2, we have that for every realization of  $\omega$ , the operating profits of a firm decrease if its competitor's debt level increases. Hence, the probability that the firm will be able to fully repay its debt decreases, as do the profits that debtholders can capture in case the firm cannot fully repay its debt.

The effect of an increase in a firm's debt level on its own debtholders' expected repayment is ambiguous. Yet, we can derive a weaker result that is sufficient for our purposes. We will refer to the common debt level that maximizes debtholder expected revenues as  $d_R$ . If this value is not unique, we define  $d_R$  as the smallest  $d$  for which the maximum of  $R(d, d)$  is reached. Thus

$$d_R \equiv \min \left\{ d : d \in \arg \max_d R(d, d) \right\}. \quad (7)$$

Suppose that a firm's competitor has a debt level that is equal to  $d_R$ , while the firm itself has a lower debt level. In that case, if we increase the amount of debt of this firm to  $d_R$  as well, we can show that the expected revenues of the debtholders of this firm increase:

**Lemma 5** *We have  $R_i(d_i, d_R) < R_i(d_R, d_R)$  for all  $d_i < d_R$ .*

## 3.2 Equilibrium

We now turn to the equilibrium of the model. In the appendix, we prove the following results:

**Theorem 1** *An equilibrium of our model is given by:*

1.  $B_k^* = b_R \equiv \max_d R(d, d)$ , for  $k = 1, \dots, N$ ,
2.  $d_i^* = d_R \equiv \min \{d : d \in \arg \max_d R(d, d)\}$ , for  $i = 1, 2$ ,
3.  $a_i^* = \arg \max_{a_i} \Pi_i(a_i, a_j^*, d_i^*)$ , for  $i = 1, 2, j \neq i$ .

**Theorem 2** *In the equilibrium described in Theorem 1, firms make strictly positive expected net profits.*

In a standard auction with symmetric bidders and full information, profits are competed away in Bertrand-like fashion. In this auction, however, this is not true. Once the bidding reaches  $b_R$ , bids will no longer increase. Suppose that a firm does submit a bid higher than  $b_R$ . To finance such a bid, the debt this firm has to take on is necessarily higher than  $d_R$ . But more debt induces a firm to compete more aggressively on the product market, which implies that the expected repayment to debtholders is lower. With higher debt, but a lower expected repayment, expected debtholder profits would then decrease. Since these are zero in  $(b_R, d_R)$ , this implies that any bidder outbidding  $b_R$  will not be able to find financing.

In effect, we thus have a backward-bending credit supply curve.<sup>8</sup> In general, such a curve may be caused by adverse selection or moral hazard. With moral hazard, as the interest rate  $r$  increases, firms are inclined to take on riskier projects. At some point, this effect becomes so strong that the expected repayment to debtholders decreases with an increase in  $r$ , causing the supply of credit to decrease. A similar argument applies in our

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<sup>8</sup>See e.g. Freixas and Rochet (1998, section 5.2).

model. As the winning firms take on more debt, they compete more aggressively, which implies that expected operating profits decrease, at some point leading to a lower expected repayment to debtholders. The reason that in our model expected net profits are positive thus boils down to credit rationing. The equilibrium of our model occurs at the maximum of the backward bending credit supply curve. Submitting higher bids would require a higher expected repayment to debtholders, which is simply not feasible. Hence firms are still willing to submit higher bids, but are not able to do so since they are denied access to the credit market when they do.

We can also show:

**Theorem 3** *The equilibrium described in Theorem 1 is the unique symmetric equilibrium, provided that the maximizer of  $R(d, d)$  is unique. If that is not the case, we still have that the equilibrium of the auction stage is unique.*

Hence, provided that the condition in the theorem is satisfied, our model has a unique symmetric equilibrium, in which all firms submit the bid. The winning bidders choose the same debt level. That debt level is the common debt level that maximizes expected repayment to the debtholders. Yet, debtholders' net profits are driven to zero. At the auction stage, firms will increase their bids as long as they are still able to obtain financing, that is, up to the point where expected net profits of debtholders are zero.

To get some intuition for this result, refer to Figure 1. Here, we depict a firm's expected operating profits  $E_\omega(\pi^*(d, d))$  as a function of the common debt level  $d$ . Note that  $E_\omega(\pi^*)$  is decreasing in  $d$ , using Lemma 3. Out of expected operating profits, the amount  $R$  flows to the debtholders. The firm is left with expected net firm profits  $\Pi$ , which equals the difference between  $E_\omega(\pi^*)$  and  $R$ . From the definition of  $\tilde{d}$ , we have  $E_\omega[\pi^*(\tilde{d}, \tilde{d}, \omega)] = R(\tilde{d}, \tilde{d})$ . As firms' behavior does not change beyond  $\tilde{d}$ , we have that for all  $d > \tilde{d}$ ,  $E_\omega[\pi^*(d, d, \omega)] = R(d, d) = E_\omega[\pi^*(\tilde{d}, \tilde{d}, \omega)]$ .

In the Figure,  $R$  is decreasing for  $d$  slightly smaller than  $\tilde{d}$ , which implies that  $d_R < \tilde{d}$ . Intuitively, this can be seen as follows. Suppose that we have  $d = \tilde{d}$ . In that case, all

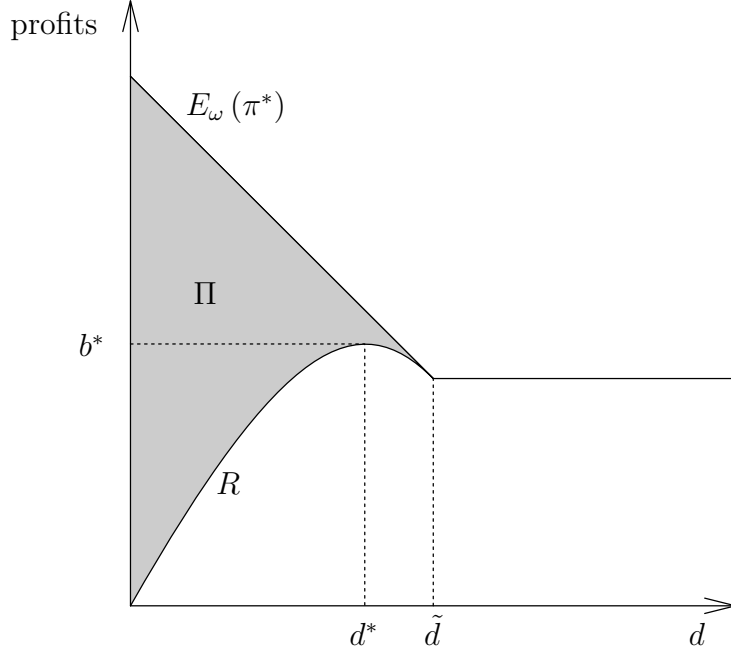


Figure 1: Solving the model.

operating profits flow to the debtholders. Now suppose that we decrease  $d$  slightly to  $\tilde{d} - \varepsilon$ . Operating profits then increase for all possible realizations of  $\omega$ . This is good news for debtholders.<sup>9</sup> Hence,  $R$  then increases.

The equilibrium of our model is now easy to see. First note that all firms submit the same bid in equilibrium. Suppose that all firms bid some  $b_0 < b_R$ . The two winning firms are then able to find financing, since there is a  $d_0$  such that  $R(d_0, d_0) = b_0$ . Yet, this cannot be an equilibrium. Suppose that one firm increases its bid to some  $b_1 = b_0 + \varepsilon < b_R$ . A firm is willing to defect in such manner, since it then wins the auction with certainty, whereas the expected profits upon winning are hardly affected. Also, both winning firms are still able to find financing: for example, by both taking on debt  $d_R$ . Since  $R(d_R, d_R) = b_R > b_1 > b_0$ , debtholders are willing to sign such contracts. The equilibrium thus necessarily has all bidders bidding  $b_R$ , and the winners taking on debt levels  $d_R$ .

<sup>9</sup>On the other hand, for  $\omega$  such that  $\omega < \hat{\omega}$ , bondholders no longer receive *all* operating profits. For very small  $\omega$ , this implies that they now receive less than in the previous case, but this is only a second-order effect.



## 4 Comparison with other allocation mechanisms

In this section we discuss the implications of our model for the outcome of the auction, and for the competitive outcome that results. In particular, we analyze the fees paid at the auction and the consumer price level in the equilibrium of our model. We compare fees and prices to those in alternative setups, such as a beauty contest or a standard auction without external financing.

We first argue that consumer prices are decreasing in the amount of debt of the firms competing on the market.

**Proposition 1** *In the competition subgame, if both firms have the same debt level, and if that debt level increases, then equilibrium consumer prices decrease:*

$$\frac{dp^*(d, d)}{dd} < 0.$$

**Proof.** This has been shown in the proof of Lemma 3. ■

Intuitively, debt induces firms to compete more aggressively. Hence, higher debt implies lower prices. (See e.g. Brander and Lewis, 1986). Now consider the case of a beauty contest, in which licenses are allocated through some administrative mechanism, at a fee that equals zero.<sup>10</sup> We still assume that firms do not have internal funds.

**Corollary 1** *A beauty contest leads to higher consumer prices than the auction with external financing does.*

**Proof.** With a beauty contest,  $d = 0$ . In the equilibrium of the auction with external financing,  $d = d_R > 0$ . The result then follows from Proposition 1. ■

In the equilibrium of our model the debt level  $d_R$  is strictly positive, whereas with a beauty contest licenses are given away for free and  $d = 0$ . Since firms compete more aggressively when they hold debt, consumer prices are lower when licenses are auctioned than when

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<sup>10</sup>Note that in general, a beauty contest can be considered as an all-pay auction, were each participant has to incur some cost for preparing his application. We ignore such costs.

they are given away for free in a beauty contest. Note that this result is opposite to what is often argued in the popular press: that auctions lead to higher consumer prices, since firms have to "earn their money back". That argument boils down to a sunk-cost fallacy (see e.g. Klemperer, 2002a). In our model, auctions lead to *lower* prices than beauty contests: auctions lead to higher debt, and higher debt leads to lower prices.

Our results depend crucially on the strictly positive debt levels that result from the stage in which the licenses are awarded. For comparison, we now discuss what happens when firms do have sufficient internal funds.

**Corollary 2** *When firms have access to sufficient internal funds, license fees paid at an auction will be higher than when firms have to resort to external finance. Consumer prices will also be higher.*

**Proof.** In appendix. ■

The intuition is straightforward. The strategic effect of debt implies that firms that have debt compete more aggressively than firms that do not. As a result, their expected operating profits in equilibrium will be lower. With internal funds, expected operating profits are competed away in the auction. Hence, equilibrium bids equal  $E_\omega(\pi^*(0, 0))$ . With positive debt levels, equilibrium bids are necessarily lower, as expected operating profits are lower. Moreover, the firms' expected profits are not competed away completely. For both reasons, equilibrium bids in a model with internal funds are higher than those in a model with debt. Also, equilibrium prices are higher. The intuition for that result is the same as in the case of a beauty contest.

## 5 Some numerical examples

In this section, we give some numerical examples. We consider a Hotelling, a Bertrand, and a Cournot model with cost uncertainty, and a Cournot model with demand uncertainty. For each case, we first show that Assumptions 1 through 3 are satisfied. Then we solve

for the equilibrium of our model with debt, and of the model in which winning bids are financed through internal funds.

### 5.1 Hotelling competition with cost uncertainty

Suppose that the two winning firms are located on a Hotelling line of unit length, one firm being located at 0, the other at 1. A mass of consumers, normalized to 1, is uniformly distributed on the line. Transportation costs are normalized to 1 per unit of distance. The willingness to pay is  $v$  for every consumer, with  $v$  high enough so the market is always covered. Assume that marginal costs  $c$  are constant and equal across firms, and drawn from a uniform distribution on  $[0, 1]$ . Obviously, operating profits are lower if marginal costs turn out to be higher. We can thus interpret  $c$  as the random variable  $\omega$  in our analysis. We have  $f(\omega) = 1$  and  $[\underline{\omega}, \bar{\omega}] = [0, 1]$ . Operating profits of firm  $i$  now equal

$$\pi_i(p_i, p_j, \omega) = \frac{1}{2}(1 + p_j - p_i)(p_i - \omega). \quad (8)$$

Note that

$$\frac{\partial^2 \pi_i(p_i, p_j, \omega)}{\partial p_i \partial \omega} = 1/2,$$

which implies that Assumption 3 is satisfied. Also

$$\hat{\omega}_i = p_i - \frac{2d_i}{1 + p_j - p_i}.$$

In stage 3, firm  $i$ 's expected net operating profits equal

$$\Pi_i = \int_0^{\hat{\omega}_i} \left( \frac{1}{2} (1 + p_j - p_i) (p_i - \omega) - d_i \right) d\omega.$$

Plugging in  $\hat{\omega}_i$  and  $\pi_i(p_i, p_j, \omega)$ , this yields

$$\Pi_i(p_i, p_j, d_i) = \frac{(p_i (1 + p_j - p_i) - 2d_i)^2}{4 (1 + p_j - p_i)}. \quad (9)$$

Taking the first-order condition yields four possible solutions for  $p_i$ :

$$\begin{aligned} p_i^1 &= \frac{1}{2}(1 + p_j) + \frac{1}{2}\sqrt{(1 + p_j)^2 - 8d_i}, \\ p_i^2 &= \frac{1}{2}(1 + p_j) - \frac{1}{2}\sqrt{(1 + p_j)^2 - 8d_i}, \\ p_i^3 &= \frac{5}{6}(1 + p_j) + \frac{1}{6}\sqrt{(1 + p_j)^2 + 24d_i}, \\ p_i^4 &= \frac{5}{6}(1 + p_j) - \frac{1}{6}\sqrt{(1 + p_j)^2 + 24d_i}. \end{aligned}$$

Yet, plugging either  $p_i^1$  and  $p_i^2$  back into the numerator of (9) yields zero profits, which implies that these roots are not feasible. Note also that  $1 + p_j - p_i^3 = \frac{1}{6}(1 + p_j) - \frac{1}{6}\sqrt{(1 + p_j)^2 + 24d_i}$ , which implies that when using  $p_i^3$  the denominator of (9) becomes negative, which implies negative profits. Therefore,  $p_i^4$  is the only relevant solution. From (8), we have that, evaluated in equilibrium,

$$\frac{d\pi_i}{dd_j} = \frac{1}{2}(1 + p_j^* - p_i^*)\frac{dp_i^*}{dd_j} + \frac{1}{2}(p_i^* - \omega)\left(\frac{\partial p_j^*}{\partial d_j} - \frac{dp_i^*}{dd_j}\right). \quad (10)$$

In this case, we have price competition. The proof of Lemma 2 then implies that both  $p_i^*$  and  $p_j^*$  are decreasing in  $d_j$ . Also note that  $d_j$  does have a direct effect on the reaction function of firm  $j$ , but not on that of firm  $i$ . We thus have

$$\frac{dp_i^*}{dd_j} = \frac{\partial p_i^*}{\partial p_j} \frac{\partial p_j^*}{\partial d_j} = \left( \frac{1}{6} \frac{5\sqrt{(1 + p_j)^2 + 24d_i} - (1 + p_j)}{\sqrt{(1 + p_j)^2 + 24d_i}} \right) \frac{\partial p_j^*}{\partial d_j} < \frac{5}{6} \frac{\partial p_j^*}{\partial d_j}.$$

This implies that the large bracketed term on the right-hand side of 10 is strictly positive. Since we require that  $\pi_i^* > 0$  for all  $\omega$ , we have  $p_i^* > \omega$ . This implies that Assumption 1 is satisfied.

Suppose both firms have the same level of debt  $d$ . We can then solve for equilibrium prices to find

$$p^*(d, d) = 2 - 2d$$

and

$$\pi^*(d, d) = \frac{1}{2}(2 - 2d - \omega).$$

This implies that  $d\pi_i^*/dd < 0$  for all  $\omega$ , so Assumption 2 is satisfied as well.

For the expected repayment to the debtholders, we have

$$R_i(d_i, d_j) = \Pr(\omega \leq \hat{\omega})d_i + \int_{\hat{\omega}_i}^{\bar{\omega}} \pi_i^*(d_i, d_j, \omega) f(\omega) d\omega = \hat{\omega}_i d_i + \int_{\hat{\omega}_i}^1 \pi_i^*(d_i, d_j, \omega) d\omega.$$

With common  $d$ , we have

$$\hat{\omega} = p^* - 2d = 2 - 4d.$$

Note that the highest possible value for  $\omega$  is 1. Hence, when  $d \leq 1/4$ , the firms are always able to repay their debt. The restriction that  $\pi_i^* > 0$  requires  $d < 1/2$ . We thus have:

$$\begin{aligned} R(d, d) &= \begin{cases} d & \text{if } d \leq \frac{1}{4} \\ (2 - 4d)d + \int_{2-4d}^1 \frac{1}{2}(2 - 2d - \omega) d\omega & \text{if } \frac{1}{4} < d < \frac{1}{2} \end{cases} \\ &= \begin{cases} d & \text{if } d \leq \frac{1}{4} \\ 3d - 4d^2 - \frac{1}{4} & \text{if } \frac{1}{4} < d < \frac{1}{2}. \end{cases} \end{aligned}$$

Maximizing with respect to  $d$  yields  $d^* = \frac{3}{8}$ ,  $b^* = R(d^*, d^*) = \frac{5}{16}$ , and  $p^* = p^*\left(\frac{3}{8}, \frac{3}{8}\right) = \frac{5}{4}$ . Firms earn strictly positive expected profits, which equal  $\frac{1}{16}$ . In a model with internal funds, equilibrium consumer prices are  $p^*(0, 0) = 2$ , which is higher than prices in a model with debt. The same consumer prices would prevail with a beauty contest. With internal funds, bids equal  $b^{int} = E_\omega(\pi^*(0, 0)) = \frac{3}{4}$  which is higher than in the model with debt, and expected firm profits are driven to zero.

## 5.2 Bertrand competition with cost uncertainty

Now consider a model of Bertrand competition among two firms,  $i = 1, 2$ , producing heterogeneous goods. Inverse demand for firm  $i$ 's product is given by

$$p_i = 5 - q_i - \frac{1}{2}q_j,$$

Demand can be written in direct form as

$$q_i = \frac{10}{3} + \frac{2}{3}p_j - \frac{4}{3}p_i$$

Assume that marginal costs  $c$  are constant and identical across firms, and drawn from a uniform distribution on  $[0, 1]$ . We again interpret  $c$  as the random variable  $\omega$  in our analysis. Operating profits of firm  $i$  are given by

$$\pi_i(p_i, p_j, c) = \frac{1}{3} (10 + 2p_j - 4p_i) (p_i - \omega). \quad (11)$$

To have  $\pi_i^* > 0$  for all  $\omega$ , we thus need  $p_i^* > 1$ . We also have

$$\frac{\partial \pi_i(p_i, p_j, \omega)}{\partial p_i \partial \omega} = \frac{4}{3} > 0.$$

Moreover,

$$\hat{\omega}_i = p_i - \frac{3}{2} \frac{d_i}{5 + p_j - 2p_i}.$$

Now

$$\Pi_i = \int_0^{\hat{\omega}_i} \left( \frac{2}{3} (5 + p_j - 2p_i) (p_i - \omega) - d_i \right) d\omega.$$

Plugging in  $\hat{\omega}_i$ , we find

$$\Pi_i = \frac{1}{12} \frac{(2p_i(5 + p_j - 2p_i) - 3d_i)^2}{5 + p_j - 2p_i}.$$

Again, taking the first-order condition yields four possible solutions. With the exact same arguments as in the case of Hotelling competition, we can eliminate three, leaving us with

$$p_i = \frac{5}{12} (5 + p_j) - \frac{1}{6} \sqrt{\left( \frac{5}{2} + \frac{1}{2} p_j \right)^2 + 9d_i}.$$

From (11), we have that, evaluated in equilibrium,

$$\frac{d\pi_i}{dd_j} = \frac{1}{3} (10 + 2p_j^* - 4p_i^*) \frac{dp_i^*}{dd_j} + \frac{2}{3} (p_i^* - \omega) \left( \frac{\partial p_j^*}{\partial d_j} - 2 \frac{dp_i^*}{dd_j} \right). \quad (12)$$

With price competition, the proof of Lemma 2 implies that both  $p_i^*$  and  $p_j^*$  are decreasing in  $d_j$ . Also note that  $d_j$  does have a direct effect on the reaction function of firm  $j$ , but not on that of firm  $i$ . We thus have

$$\frac{dp_i^*}{dd_j} = \frac{\partial p_i^*}{\partial p_j} \frac{\partial p_j^*}{\partial d_j} = \frac{1}{12} \left( \frac{5\sqrt{(5 + p_j^*)^2 + 36d_i} - (5 + p_j)}{\sqrt{(5 + p_j)^2 + 36d_i}} \right) \frac{\partial p_j^*}{\partial d_j} < \frac{5}{12} \frac{\partial p_j^*}{\partial d_j}.$$

Since we require that  $\pi_i^* > 0$  for all  $\omega$ , we have  $p_i^* > \omega$ . This implies, from (12), that Assumption 1 is satisfied.

Consider the case in which both firms face the same debt  $d$ . Imposing symmetry,

$$p^*(d, d) = \frac{1}{4} \left( 15 - \sqrt{25 + 12d} \right)$$

so

$$\hat{\omega} = p^* - \frac{3d}{2(5 - p^*)} = \frac{25}{4} - \frac{3}{4} \sqrt{12d + 25}.$$

Equilibrium profits now equal

$$\pi^*(d, d, \omega) = \frac{1}{12} \left( \left( \sqrt{12d + 25} + 5 \right) (5 - 2\omega) - 6d \right).$$

Therefore, when  $d < 2$ , we have  $\pi^*(d, d, \omega) > d$ , so the firm is always able to repay its debt.

The necessary condition that  $p^* > 1$  implies  $d < 8$ . We have

$$\frac{d\pi^*}{dd} = \frac{1}{2} \left( \frac{5 - 2\omega}{\sqrt{12d + 25}} - 1 \right),$$

which is negative for all  $\omega \in [0, 1]$  and  $d \geq 0$ . Hence, Assumption 2 is satisfied.

For the expected repayment to the debtholders, we have

$$\begin{aligned} R(d, d) &= \begin{cases} d & \text{if } d \leq 2 \\ \int_{\frac{25}{4} - \frac{3}{4}\sqrt{12d+25}}^1 \left( \frac{1}{12} \left( (\sqrt{12d+25} + 5) (5 - 2\omega) - 6d \right) \right) d\omega & \text{if } 2 < d < 8 \end{cases} \\ &= \begin{cases} d & \text{if } d \leq 2 \\ \frac{97}{16}d - \frac{15}{16} - \frac{3}{16} (\sqrt{12d+25}) (3d+1) & \text{if } 2 < d < 8. \end{cases} \end{aligned}$$

Maximizing with respect to  $d$  yields  $d^* = \frac{194}{2187} \sqrt{907} + \frac{1424}{2187} \approx 3.3226$ ,  $b^* = R(d^*, d^*) = \frac{1814}{6561} \sqrt{907} - \frac{37294}{6561} \approx 2.6425$ , and  $p^* = \frac{1}{4} \left( \frac{308}{27} - \frac{4}{27} \sqrt{907} \right) \approx 1.7364$ . Firms earn strictly positive expected profits, which equal approximately 0.0476. In a model with internal funds, equilibrium consumer prices are  $p^*(0, 0) = \frac{5}{2}$ , which is higher than prices in a model with debt. The same consumer prices would prevail with a beauty contest. With internal funds, bids equal  $b^{int} = E_\omega(\pi^*(0, 0)) = \frac{10}{3}$ , which is higher than in a model with debt, and expected firm profits are driven to zero.

### 5.3 Cournot competition with cost uncertainty

Consider quantity competition among the two firms,  $i = 1, 2$ . Suppose inverse demand is

$$p = 3 - q_1 - q_2. \quad (13)$$

Again, we assume uncertainty about a common and fixed level of marginal cost  $c$ , so  $\omega \equiv c$ .

Moreover, we again assume  $f(\omega) = 1$  on  $[0, 1]$ . Operating profits then are

$$\pi_i(q_i, q_j, \omega) = (3 - q_i - q_j - \omega) q_i \quad (14)$$

for  $i, j = 1, 2$ ,  $i \neq j$ . From this expression,

$$\hat{\omega}_i = 3 - q_i - q_j - \frac{d_i}{q_i}.$$

Now

$$\begin{aligned} \Pi_i &= \int_0^{\hat{\omega}_i} ((3 - q_i - q_j - \omega) q_i - d_i) d\omega \\ &= \frac{1}{2q_i} (q_i (3 - q_i - q_j) - d_i)^2. \end{aligned}$$

Again, taking the first-order condition yields four possible solutions, three of which can be eliminated. The relevant solution is

$$q_i = \frac{1}{2} - \frac{1}{6}q_j + \frac{1}{6}\sqrt{(3 - q_j)^2 + 12d_i}.$$

From (14), we have that, evaluated in equilibrium,

$$\frac{d\pi_i}{dd_j} = (3 - q_i^* - q_j^* - \omega) \frac{dq_i^*}{dd_j} - \left( \frac{dq_i^*}{dd_j} + \frac{\partial q_j^*}{\partial d_j} \right) \cdot q_i^* \quad (15)$$

With quantity competition, the proof of Lemma 2 implies that  $q_i^*$  is decreasing, and  $q_j^*$  is increasing in  $d_j$ . Also note that  $d_j$  does have a direct effect on the reaction function of firm  $j$ , but not on that of firm  $i$ . We thus have:

$$\frac{dq_i^*}{dd_j} = \frac{\partial q_i^*}{\partial q_j} \frac{\partial q_j^*}{\partial d_j} = -\frac{1}{6} \left( \frac{\sqrt{(3 - q_j)^2 + 12d_i} + (3 - q_j)}{\sqrt{(3 - q_j)^2 + 12d_i}} \right) \frac{\partial q_j^*}{\partial d_j} < -\frac{1}{6} \frac{\partial q_j^*}{\partial d_j}.$$



Since we require that  $\pi_i^* > 0$  for all  $\omega$ , we have  $p_i^* > \omega$ . This implies that, from (13) and (15), we have that Assumption 1 is satisfied.

When firms face the same debt  $d$ , we have

$$q^*(d, d) = \frac{3}{8} + \frac{1}{8}\sqrt{9 + 16d}.$$

To have  $\pi^* > 0$  for all  $\omega$ , we need  $3 - 2q^* - 1 > 0$ , hence  $d < 1$ . Further note

$$\pi^*(d, d, \omega) = \frac{1}{16} \left( (3 - 2\omega) \left( \sqrt{16d + 9} + 3 \right) - 8d \right),$$

hence

$$\frac{d\pi^*}{dd} = \frac{1}{2} \left( \frac{3 - 2\omega}{\sqrt{9 + 16d}} - 1 \right),$$

which is strictly negative for all  $\omega \in [0, 1]$  and  $d \geq 0$ . Hence, Assumption 2 is satisfied.

It is easy to see that with  $d \leq 5/18$ , the firms are always able to repay their debt. Debtholder revenues can be shown to equal

$$R(d, d) = \begin{cases} d & \text{if } d \leq \frac{5}{18} \\ \frac{1}{64} (220d - 3 - (\sqrt{16d + 9})(36d + 1)) & \text{if } \frac{5}{18} < d < 1. \end{cases}$$

Maximizing with respect to  $d$  yields  $d^* \simeq 0.80818$ ,  $b^* = R(d^*, d^*) \simeq 0.52915$ , and  $q^* \simeq 0.96038$ . Firms earn strictly positive expected profits, which equal approximately 0.0271. In a model with internal funds, equilibrium quantities are  $q^*(0, 0) = \frac{3}{4}$ , which is lower than quantities in a model with debt, and hence prices will be higher than in a model with debt. With internal funds, bids equal  $b^{int} = E_\omega(\pi^*(0, 0)) = \frac{3}{4}$ , which is higher than in a model with debt, and expected firm profits are driven to zero.

## 5.4 Cournot competition with demand uncertainty

Consider quantity competition among the two firms,  $i = 1, 2$ . Suppose inverse demand is

$$p = 2 + \alpha - q_i - q_j,$$

Assume marginal costs are zero and  $f(\alpha) = 1$  on  $[0, 1]$ . Operating profits then are

$$\pi_i(q_i, q_j, \alpha) = (2 + \alpha - q_i - q_j) q_i$$

$i, j = 1, 2, i \neq j$ . Note that an increase in  $\alpha$  increases profits. Therefore, we define  $\omega \equiv 1 - \alpha$ : higher values of  $\omega$  then imply lower profits, as our analysis requires. The profit function then becomes

$$\pi_i(q_i, q_j, \omega) = (3 - \omega - q_i - q_j) q_i.$$

Note that this problem is analytically identical to the one we considered in the previous example. Hence, also in this example, all our assumptions are satisfied, and we obtain the same equilibrium as in our example with Cournot competition and cost uncertainty.

## 6 An alternative specification

Above, we solved our model for the case in which more debt induces firms to compete more aggressively. Yet, there may also be cases in which more debt induces firms to compete *less* aggressively. Showalter (1995) shows that this is the case with Bertrand competition with differentiated products, and uncertainty about demand. It is interesting to see how this affects the outcome of our model. We study that issue in this section, and thus replace Assumption 3 with the following:

**Assumption 4** *Marginal profit is strictly increasing in  $\omega$ . That is,  $\frac{\partial^2 \pi_i}{\partial \omega \partial q_i} > 0$ .*

We will discuss our results solely in terms of price competition, as we are not aware of any models with quantity competition for which Assumption 4 is satisfied. All other assumptions are maintained. We now have the following lemmas. The proofs of Lemmas 6 through 9 are straightforward variations on the equivalent lemmas in the analysis above, and are therefore omitted. All other proofs are in the appendix. In each lemma and theorem that follows, we assume that Assumptions 1, 2, and 4 are satisfied.

First we have that a firm's reaction curve shifts outwards as its level of debt increases:

**Lemma 6** *Having more debt induces a firm to compete less aggressively. That is,*

$$\frac{d\beta_i}{dd_i} \begin{cases} > 0 & \text{if } d_i \in [0, \tilde{d}_i(d_j)] \\ = 0 & \text{if } d_i > \tilde{d}_i(d_j). \end{cases}$$

This causes a firm's operating profits to increase as its competitor's debt level increases:

**Lemma 7** *An increase in a firm's debt level increases the competitor's equilibrium operating profits:*

$$\frac{\partial \pi_i^*(d_i, d_j, \omega)}{\partial d_j} \begin{cases} > 0 & \text{if } d_i \in [0, \tilde{d}_i(d_j)], \\ = 0 & \text{otherwise,} \end{cases} \quad \forall \omega \in [\underline{\omega}, \bar{\omega}].$$

The effect of an increase in the common debt level is also opposite to that in the previous section:

**Lemma 8** *An increase in the common debt level increases the expected operating profits of the firms:*

$$\frac{dE_\omega(\pi^*(d, d))}{dd} \begin{cases} > 0 & \text{if } d \in [0, \tilde{d}], \\ = 0 & \text{otherwise.} \end{cases}$$

We also have:

**Lemma 9** *An increase in the amount of debt that a firm holds, increases the expected revenue for debtholders of the other firm:*

$$\frac{\partial R_i(d_i, d_j)}{\partial d_j} \begin{cases} > 0 & \text{if } d_i \in [0, \tilde{d}_i(d_j)], \\ = 0 & \text{otherwise.} \end{cases}$$

The intuition is very similar to that in section 3. From Lemma 7, we now have that for every realization of  $\omega$ , the operating profits of the other firm increase in  $d_j$ . Hence, the probability that that firm will be able to repay its debt increases, as do the profits that debtholders can capture if the firm cannot fully repay its debt.

In this case,  $R(d, d)$  is strictly increasing on the relevant interval:

**Lemma 10** *Debtholder's revenue is strictly increasing in the common debt level:*

$$\frac{dR_i(d, d)}{dd} \begin{cases} > 0 & \text{if } d \in [0, \tilde{d}], \\ = 0 & \text{otherwise.} \end{cases}$$

This lemma has some important implications. For example, it immediately implies that  $d_R = \tilde{d}$ . For the equilibrium of our model, we again have:

**Theorem 4** *An equilibrium of our model is given by*

1.  $B_k^* = b_R \equiv \max_d R(d, d)$ , for  $k = 1, \dots, N$ ,
2.  $d_i^* = d_R \equiv \min \{d : d \in \arg \max_d R(d, d)\}$ , for  $i = 1, 2$ ,
3.  $a_i^* = \arg \max_{a_i} \Pi_i(a_i, a_j^*, d_i^*)$ , for  $i = 1, 2, j \neq i$ .

However, we now have:

**Theorem 5** *In the equilibrium described in Theorem 4, firms make zero expected net profits.*

**Theorem 6** *The equilibrium described in Theorem 4 is the unique symmetric equilibrium.*

Hence, we again have a unique symmetric equilibrium of the auction stage. All firms submit the same bid, and the firms that win the auction choose the same debt level. That debt level is a common debt level that maximizes expected repayment to the debtholders. Yet, different from the previous case, debtholders' profits are driven to zero. At the auction stage, firms will increase their bids as long as they are still able to obtain financing, that is, up to the point where expected net profits of debtholders are zero.

The outcome is depicted in Figure 2. Again, a firm's expected operating profits  $E_\omega(\pi^*(d, d))$  are depicted as a function of a common debt level  $d$ . Now we have that  $E_\omega(\pi^*)$  is increasing in  $d$ , using Lemma 8. Out of expected operating profits, the amount  $R$  flows to debtholders. The firm is left with expected net firm profits  $\Pi$ , which equal the difference between  $E_\omega(\pi^*)$  and  $R$ . From the definition of  $\tilde{d}$ , we have  $E_\omega[\pi^*(\tilde{d}, \tilde{d}, \omega)] = R(\tilde{d}, \tilde{d})$ . As firms' behavior does not change beyond  $\tilde{d}$ , we have that for all  $d > \tilde{d}$ ,  $E_\omega[\pi^*(d, d, \omega)] = R(d, d) = E_\omega[\pi^*(\tilde{d}, \tilde{d}, \omega)]$ .

We now have that  $R$  is increasing for all  $d < \tilde{d}$ , which implies that  $d_R = \tilde{d}$ . Intuitively, this can be seen as follows. Suppose that we have  $d = \tilde{d}$ . When we decrease  $d$  from  $\tilde{d}$  to some lower value, firms will compete more aggressively. This hurts debtholders. And even

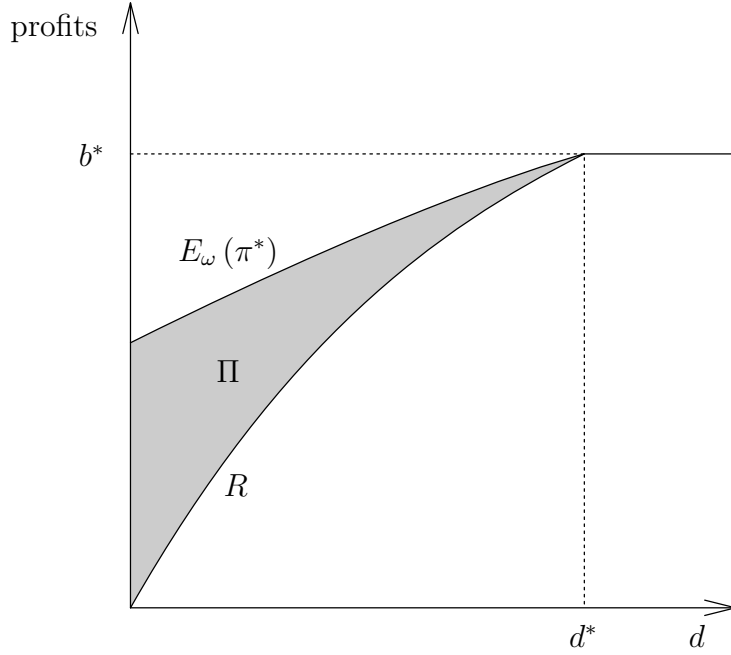


Figure 2: Solving the model for the alternative specification.

if firms are able to repay their debt, debtholders receive a lower amount. Hence,  $R$  then decreases. Again, in equilibrium we must have that both firms submit  $b_R$ , with the same argument as in the previous case. But now, since  $d_R$  coincides with  $\tilde{d}$ , it no longer implies that firms have positive expected profits. There is no longer a moral hazard problem, as higher debt levels only increase equilibrium firm profits. Therefore, we no longer have credit rationing.

It is easy to see that higher debt levels now imply higher consumer prices. This directly implies that, in the set-up that we have here, consumer prices will be *lower* with a beauty contest than they are with an auction with debt. Consumer prices with an auction with internal funds are equal to those with a beauty contest, and lower than those with an auction with debt. From Figure 2, it is also easy to see that auction revenues will be lower with internal funds than with debt.

Unfortunately, it is not possible to find numerical examples for this case. Solving a simple linear Hotelling model with demand uncertainty, for example, involves solving a

fourth-order polynomial, for which we are not able to find an analytical solution.

## 7 Discussion and conclusion

In this paper, we considered license auctions in which winning firms have to take on debt in order to finance their bids. Since debt has a strategic effect in the aftermarket, it will also affect the outcome of the auction. When debt induces firms to compete more aggressively, there is a negative relation between consumer prices and the fees paid at the auction. Thus, higher fees imply lower prices for consumers. Even though firms are completely symmetric, expected equilibrium profits are strictly positive. This is due to credit rationing. Winning bidders, even though they make positive expected profits, will not be outbid. Any higher bid would yield a debt level that implies negative net expected profits for debtholders, and therefore financing cannot be obtained. We also showed that auction revenues are lower if firms finance bids by taking on debt rather than through internal funds. Consumer prices are lower with an auction with debt, than they are with a beauty contest or with an auction with internal funds.

The results change, however, when debt induces firms to compete *less* aggressively. In that case, there is no credit rationing, so expected firm profits are driven to zero. Auction revenues now are higher if firms finance bids by taking on debt rather than through internal funds. Consumer prices are higher with an auction with debt than they are with a beauty contest, or with an auction with internal funds.

These results suggest that, in auction design, it is important to realize how winners will finance their bids. When external finance is used, results from standard auction theory, implicitly based on internal finance, do not necessarily apply. However, it is not straightforward to see what this implies for social welfare. In an auction with debt, prices may be lower in equilibrium, but the probability that firms go bankrupt increases.

Our model can be extended in a number of ways. One straightforward extension concerns the number of licenses that is being sold. In our model,  $N > 2$  potential entrants

compete for 2 licenses. Alternatively, they could compete for  $n$  licenses, with  $2 \leq n < N$ . On the output market,  $n$  firms would then compete. In this setup, there would still be a strategic effect of debt. However, an increase in  $n$  will imply a decrease in the bids because of lower expected operating profits, and thus a decrease in the debt level. Thus, an increase in  $n$  weakens the strategic effects of debt which may decrease firms' equilibrium profits.

It would also be interesting to look at asymmetries in the amount of internal funds that firms have, for example by looking at a case in which some firms are able to fully (or partially) finance their bids through internal funds, whereas other firms have to finance their entire bid on the credit market. One possible interpretation of such a scenario is that incumbent firms have their own funds, whereas potential entrants need debt financing.

## Appendix

**Proof of Lemma 1** Consider the case in which  $d_i \leq \tilde{d}_i(d_j)$ . The first-order condition (FOC) for firm  $i$  in the competition stage is given by

$$\frac{\partial \Pi_i(a_i, a_j, d_i)}{\partial a_i} = 0. \quad (16)$$

Totally differentiating this expression yields

$$\frac{\partial^2 \Pi_i}{\partial a_i^2} da_i + \frac{\partial^2 \Pi_i}{\partial a_j \partial a_i} da_j + \frac{\partial^2 \Pi_i}{\partial d_i \partial a_i} dd_i = 0.$$

A similar equality holds for firm  $j$ . This system of two equations can be solved using Cramer's rule to give

$$\frac{d\beta_i}{dd_i} \equiv \frac{da_i}{dd_i} = -\frac{\frac{\partial^2 \Pi_i}{\partial d_i \partial a_i} \frac{\partial^2 \Pi_j}{\partial a_j^2}}{H}, \quad (17)$$

where

$$H \equiv \frac{\partial^2 \Pi_i}{\partial a_i^2} \frac{\partial^2 \Pi_j}{\partial a_j^2} - \frac{\partial^2 \Pi_i}{\partial a_j \partial a_i} \frac{\partial^2 \Pi_j}{\partial a_i \partial a_j}.$$

Stability of the equilibrium implies that  $H > 0$  (see e.g. Tirole, 1988, p. 324, fn. 37). The second-order conditions (SOCs) are  $\partial^2 \Pi_i / \partial a_i^2 < 0$ . From (17) we then have that the sign

of  $da_i/dd_i$  is the same as the sign of  $\partial^2\Pi_i/\partial d_i\partial a_i$ . To sign this expression, consider the FOC given by (16). From (1), and dropping the arguments of  $\hat{\omega}_i$ , we have

$$\begin{aligned}\frac{\partial\Pi_i}{\partial a_i} &= \int_{\underline{\omega}}^{\hat{\omega}_i} \frac{\partial\pi_i(a_i, a_j, \omega)}{\partial a_i} f(\omega) d\omega + (\pi_i(a_i, a_j, \hat{\omega}_i) - d_i) f(\hat{\omega}_i) \frac{\partial\hat{\omega}_i}{\partial a_i} \\ &= \int_{\underline{\omega}}^{\hat{\omega}_i} \frac{\partial\pi_i(a_i, a_j, \omega)}{\partial a_i} f(\omega) d\omega = 0,\end{aligned}\tag{18}$$

where the second equality follows from the fact that

$$\pi_i(a_i, a_j, \hat{\omega}_i) = d_i.\tag{19}$$

From (18), we obtain

$$\frac{\partial^2\Pi_i}{\partial d_i\partial a_i} = \frac{\partial\pi_i(a_i, a_j, \hat{\omega}_i)}{\partial a_i} \cdot f(\hat{\omega}_i) \cdot \frac{\partial\hat{\omega}_i}{\partial d_i}.\tag{20}$$

This derivative equals the product of three terms. We consider the sign of each in turn, starting with the last one. Differentiating (19) with respect to  $d_i$  yields

$$\frac{\partial\pi_i}{\partial\hat{\omega}_i} \frac{\partial\hat{\omega}_i}{\partial d_i} = 1,$$

so  $\partial\hat{\omega}_i/\partial d_i = [\partial\pi_i(a_i, a_j, \hat{\omega}_i)/\partial\hat{\omega}_i]^{-1} < 0$ , hence the last term of (20) is negative. The second term is clearly positive:  $f(\hat{\omega}_i) > 0$ , provided  $\hat{\omega}_i \in (\underline{\omega}, \bar{\omega})$ . Finally, consider the sign of the first term,  $\partial\pi_i(a_i, a_j, \hat{\omega}_i)/\partial a_i$ . From Assumption 3, we have

$$\frac{\partial^2\pi_i}{\partial\omega\partial a_i} \begin{cases} < 0 \text{ if } a_i \equiv q_i, \\ > 0 \text{ if } a_i \equiv p_i. \end{cases}$$

Thus,  $\partial\pi_i(a_i, a_j, \omega)/\partial a_i$  is decreasing in  $\omega$  in case of quantity competition ( $a_i \equiv q_i$ ) and increasing in  $\omega$  in case of price competition ( $a_i \equiv p_i$ ). Consider the integral in (18). With quantity competition, we have that the integrand is decreasing, and the integral equals zero. This necessarily implies that the integrand is negative when evaluated in the upper limit of the integral, hence  $\partial\pi_i(a_i, a_j, \hat{\omega}_i)/\partial a_i < 0$ . From (20), we then have  $\partial^2\Pi_i/\partial d_i\partial a_i > 0$ . In turn, this implies from (17) that indeed  $da_i/dd_i > 0$ . With price competition, the integrand in (18) is increasing, while the integral is zero. This implies that the integrand is positive



when evaluated in  $\hat{\omega}_i$ , so  $\partial \pi_i(a_i, a_j, \hat{\omega}_i) / \partial a_i > 0$ . From (20), we then have  $\partial^2 \Pi_i / \partial d_i \partial a_i < 0$ .

In turn, this implies from (17) that indeed  $d\beta_i/dd_i < 0$ .

For  $d_i \geq \tilde{d}_i(d_j)$ , the result is trivial. By assumption, an increase in  $d_i$  does not affect  $a_i$ , hence it does not affect the reaction function  $\beta_i$ .

**Proof of Lemma 2** The equilibrium of the competition stage is determined by the intersection of the best-reply functions. Note that the best-reply function of firm  $i$  is derived from the maximization of  $\Pi_i(a_i, a_j, d_i)$ . First consider the case of price competition. Suppose that firm  $j$  faces an infinitesimally small increase in its debt level from  $d_j^O$  to  $d_j^N$ , so  $d_j^N > d_j^O$ , with  $d_j^O < \tilde{d}_j(d_i)$ . From Lemma 1, we have that  $\beta_j(p_i; d_j^N) < \beta_j(p_i; d_j^O)$ . Firm  $i$ 's best-reply function is left unaffected by the change in firm  $j$ 's debt level. Denote the initial equilibrium of the competition stage by  $(p_i^O, p_j^O)$ , and the new equilibrium by  $(p_i^N, p_j^N)$ . Since both best-reply functions are upward sloping, we must have  $p_i^N < p_i^O$  and  $p_j^N < p_j^O$ . Now,

$$\Pi_i(p_i^N, p_j^N, d_i) < \Pi_i(p_i^N, p_j^O, d_i)$$

since a decrease in the competitor's price decreases this firm's profits, as products are substitutes. Also,

$$\Pi_i(p_i^O, p_j^O, d_i) > \Pi_i(p_i^N, p_j^O, d_i)$$

since  $p_i^O$  is firm  $i$ 's best-reply function to  $p_j^O$  and  $p_i^N$  is not. Hence, combining inequalities,

$$\Pi_i(p_i^N, p_j^N, d_i) < \Pi_i(p_i^O, p_j^O, d_i),$$

so  $\Pi_i(d_i, d_j)$  is decreasing in  $d_j$ . Thus, using (3),

$$\begin{aligned} \frac{\partial \Pi_i(d_i, d_j)}{\partial d_j} &= \int_{\underline{\omega}}^{\hat{\omega}_i} \frac{\partial \pi_i^*(d_i, d_j, \omega)}{\partial d_j} f(\omega) d\omega + (\pi_i^*(d_i, d_j, \hat{\omega}_i) - d_i) f(\hat{\omega}_i) \frac{\partial \hat{\omega}_i}{\partial d_j} \\ &= \int_{\underline{\omega}}^{\hat{\omega}_i} \frac{\partial \pi_i^*(d_i, d_j, \omega)}{\partial d_j} f(\omega) d\omega < 0, \end{aligned} \quad (21)$$

where the second equality follows from the fact that  $\pi_i^*(d_i, d_j, \hat{\omega}_i) = d_i$ . Using Assumption 1, this implies that  $\pi_i^*(d_i, d_j, \omega)$  is decreasing in  $d_j$  for any  $\omega$ .

Now consider the quantity competition case. Suppose firm  $j$  faces an infinitesimally small increase in its debt level from  $d_j^O$  to  $d_j^N$ , with  $d_j^O < \tilde{d}_j(d_i)$ . From Lemma 1, we then have  $\beta_j(q_i; d_j^N) > \beta_j(q_i; d_j^O)$ . Firm  $i$ 's best-reply function is again left unaffected by the change in firm  $j$ 's debt level. Denote the initial equilibrium of the competition stage by  $(q_i^O, q_j^O)$ , and denote the new equilibrium by  $(q_i^N, q_j^N)$ . Since both best-reply functions are downward sloping, we must have  $q_j^N > q_j^O$  but  $q_i^N < q_i^O$ . Now,

$$\Pi_i(q_i^N, q_j^N, d_i) < \Pi_i(q_i^N, q_j^O, d_i)$$

since an increase in the competitor's quantity decreases this firm's profits. Also,

$$\Pi_i(q_i^O, q_j^O, d_i) > \Pi_i(q_i^N, q_j^O, d_i)$$

since  $q_i^O$  is firm  $i$ 's best-reply to  $q_j^O$  and  $q_i^N$  is not. Hence, combining inequalities,

$$\Pi_i(q_i^N, q_j^N, d_i) < \Pi_i(q_i^O, q_j^O, d_i),$$

so  $\Pi_i(d_i, d_j)$  is decreasing in  $d_j$ . Again, using (3) and Assumption 1, this implies that  $\pi_i^*(d_i, d_j, \omega)$  is decreasing in  $d_j$  for any  $\omega$ .

For  $d_i \geq \tilde{d}_i(d_j)$ , the result is trivial. By assumption, an increase in  $d_i$  does not affect  $a_i$ , hence it does not affect  $a_j$ , so profits will also be unaffected.

**Proof of Lemma 3** First consider the case of price competition. From Lemma 1, an increase in firm  $i$ 's debt level shifts its best-reply function downwards. Since best-reply functions are upward sloping, firms are symmetric, and  $d_i = d_j \equiv d$ , an increase in  $d$  must decrease equilibrium prices for both firms:  $dp^*(d, d)/dd < 0$ . Next, define

$$p^m \equiv \arg \max_p E_\omega(\pi(p, p)).$$

Thus,  $p^m$  is the price that, if set by both firms, maximizes their expected operating profits. Note that expected operating profits are equal to firm profits in the case of zero debt:  $E_\omega(\pi(p, p)) = \Pi(p, p, 0)$ . Using symmetry between the two firms and the condition for

stability of the Nash equilibrium, we then have that  $E_\omega(\pi(p, p))$  is increasing in  $p$  for  $p < p^m$ .

Obviously, the equilibrium price in a duopoly is strictly lower than the price that maximizes joint profits,<sup>11</sup> hence  $p^*(0, 0) < p^m = \arg \max \Pi(p, p, 0)$ . With  $dp^*(d, d)/dd < 0$ , this implies that  $p^*(d, d) < p^m$  for all  $d$ , and hence  $dE_\omega(\pi^*(d, d))/dd < 0$ .

For the case of quantity competition, we have that best-reply functions are downward sloping and, from Lemma 1, that an increase in  $d$  leads to an upward shift in a firm's best-reply function. Hence, equilibrium quantities  $q^*(d, d)$  are increasing in  $d$ :  $dq^*(d, d)/dd > 0$ . Define

$$q^m \equiv \arg \max_q E_\omega(\pi(q, q)).$$

Thus,  $q^m$  is the price that, if set by each individual firm, maximizes the firms' expected operating profits. Concavity of  $\pi(q, q)$  implies that  $E_\omega(\pi(q, q))$  is decreasing in  $q$  for  $q > q^m$ . Obviously, the equilibrium quantity a firm sets in a duopoly is strictly higher than the quantity each firm sets when joint profits are maximized<sup>12</sup>, hence  $q^*(0, 0) > q^m \equiv \arg \max \Pi_i(p, p, 0)$ . With  $dq^*(d, d)/dd > 0$ , this implies that  $q^*(d, d) > q^m$  for all  $d$ , and hence  $dE_\omega(\pi^*(d, d))/dd < 0$ .

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<sup>11</sup>Formally, note that  $p^*(0, 0)$  solves

$$\frac{\partial \Pi_i(p_i, p^*, 0)}{\partial p_i} = 0.$$

Plugging this into the FOC of joint-profit maximization yields

$$\frac{\partial \Pi_i(p^*, p^*, 0)}{\partial p_i} + \frac{\partial \Pi_i(p^*, p^*, 0)}{\partial p_j} > 0,$$

as  $\partial \Pi_i / \partial p_j > 0$ , since an increase in  $j$ 's price will increase  $i$ 's profits. Strict concavity then implies that the joint-profit maximizing price  $p^m$  is strictly higher than  $p^*$ .

<sup>12</sup>Formally, note that  $q^*(0, 0)$  solves

$$\frac{\partial \Pi_i(q_i, q^*, 0)}{\partial q_i} = 0.$$

Plugging this into the FOC of joint-profit maximization yields

$$\frac{\partial \Pi_i(q^*, q^*, 0)}{\partial q_i} + \frac{\partial \Pi_i(q^*, q^*, 0)}{\partial q_j} < 0,$$

as  $\partial \Pi_i / \partial q_j < 0$ , since an increase in  $j$ 's quantity will decrease  $i$ 's profits. Strict concavity then implies that the joint-profit maximizing quantity  $q^m$  is strictly lower than  $q^*$ .

**Proof of Lemma 4** Using Leibniz's rule, from (2) the partial derivative of  $R_i(d_i, d_j)$  with respect to  $d_j$  is given by

$$\frac{\partial}{\partial d_j} R_i(d_i, d_j) = \frac{\partial \Pr(\omega \leq \hat{\omega}_i)}{\partial d_j} d_i - \pi_i^*(d_i, d_j, \hat{\omega}_i) f(\hat{\omega}_i) \frac{\partial \hat{\omega}_i}{\partial d_j} + \int_{\hat{\omega}_i}^{\bar{\omega}} \frac{\partial \pi_i^*(d_i, d_j, \omega)}{\partial d_j} f(\omega) d\omega.$$

Since  $\pi_i^*(d_i, d_j, \hat{\omega}_i) = d_i$  and because  $f(\omega)$  is the probability density function of  $\omega$ , the first and second terms cancel out. Thus, only the third term remains. This term is negative because of Lemma 2. For  $d_j > \tilde{d}_j$ , a change in  $d_j$  does not affect  $a_j^*$ , which implies that  $R_i(d_i, d_j)$  is also unaffected.

**Proof of Lemma 5** Since  $d_i < d_R$ , we have from (7) that  $R_i(d_i, d_i) < R_i(d_R, d_R)$ , and from Lemma 4 that  $R_i(d_i, d_R) < R_i(d_i, d_i)$  since  $d_R \leq \tilde{d}$  by construction. Hence  $R_i(d_i, d_R) < R_i(d_R, d_R)$ .

**Proof of Theorem 1** To prove that this is an equilibrium, we need that the three conditions in definition 1 are satisfied. Equilibrium at the competition stage holds by definition for the  $a_i^*$  defined in the Theorem. For the debt stage, we need to show that  $(d_R, d_R)$  is an equilibrium, given that both firms have submitted a bid  $b_R$  in the auction stage. For the auction stage, we need to show that no firm is willing to submit a higher bid and that firms submitting  $b_R$  make nonnegative profits.

Consider the *debt stage*. Equilibrium debts  $(d_1, d_2)$  have to satisfy

$$R_1(d_1, d_2) \geq b_R, \tag{22}$$

and

$$R_2(d_2, d_1) \geq b_R. \tag{23}$$

Consider firm 1's best reply to  $d_2 = d_R$ . With  $d_1 < d_R$ , we have from Lemma 5 that  $R_1(d_1, d_2) < R_1(d_R, d_R) = b_R$ , contradicting (22). Consider the possibility to set  $d_1 > d_R$ . With  $d_R < \tilde{d}(d_R)$ , we have from Lemma 4 for any  $d_1 > d_R$  that  $R_2(d_R, d_1) < R_2(d_R, d_R)$  contradicting (23). With  $d_R = \tilde{d}(d_R)$ , profits for firm 1 would be zero for any  $d_1 \geq d_R$ ,

hence firm 1 would be indifferent between setting any  $d_1 \geq d_R$ , so (22) and (23) hold with equality. With  $d_1 = d_R$ , we also have that both (22) and (23) hold with equality. Hence  $(d_R, d_R)$  is an equilibrium for the debt stage.

Now consider the *auction stage*. First note that in the equilibrium suggested in Theorem 1, trivially, a firm cannot make negative profits if all firms submit  $b_R$ ; firms will be able to find financing, and limited liability in the competition stage precludes negative profits. Hence, a firm cannot strictly improve by submitting a lower bid and losing the auction. Consider a firm that defects by submitting a bid  $b > b_R$ , so we have some  $b_1 > b_R$  and  $b_2 = b_R$ . The conditions (6) imply that to be able to obtain financing for both firms in the new situation, we need to find a  $(d_1, d_2)$  such that

$$R_1(d_1, d_2) \geq b_1 > b_R, \quad (24)$$

and

$$R_2(d_2, d_1) \geq b_R. \quad (25)$$

The argument proceeds with the following steps:

1. *There is no such  $(d_1, d_2)$  with  $d_1 = d_2$ .* — With  $d_1 = d_2$ , we immediately have from the definition of  $d_R$  that  $R_1(d_1, d_2) \leq R_1(d_R, d_R)$ , contradicting (24).
2. *There is no such  $(d_1, d_2)$  with either  $d_1 = d_R$  or  $d_2 = d_R$ .* — Without loss of generality, assume  $d_2 = d_R$ . If  $d_1 < d_R$ , we have from Lemma 5 that  $R_1(d_1, d_R) < R_1(d_R, d_R)$ , contradicting (24). If  $d_1 > d_R$ , we consider two subcases.
  - (a) Suppose  $d_R < \tilde{d}_1(d_R)$ . In this case, a change in firm 1's debt level from  $d_R$  to  $d_1$  will have an effect on firm 1's behavior in the competition stage. From Lemma 4, we then have  $R_2(d_R, d_1) < R_2(d_R, d_R)$ , contradicting (25).
  - (b) Suppose  $d_R \geq \tilde{d}_1(d_R)$ . In this case, a change in firm 1's debt level from  $d_R$  to  $d_1$  will not have an effect on firm 1's behavior in the competition stage. Hence  $R_2(d_R, d_1) = R_2(d_R, d_R)$  and  $R_1(d_1, d_R) = R_1(d_R, d_R)$ , contradicting (24).

3. *There is no such  $(d_1, d_2)$  with  $d_1 > d_R$  and  $d_2 > d_R$ .* — Assume  $d_1 > d_2 > d_R$ . With similar arguments as those used below, we can also rule out the case  $d_2 > d_1 > d_R$ . We consider two subcases:

(a) Suppose  $d_2 < \tilde{d}_1(d_2)$ . Consider a decrease in firm 1's debt level from  $d_1$  to  $d_2$ . This will have an effect on firm 1's behavior in the competition stage. From Lemma 4, we then have  $R_2(d_2, d_1) < R_2(d_2, d_2)$ . From the definition of  $d_R$ , we have  $R_2(d_2, d_2) \leq R_2(d_R, d_R)$ . Hence  $R_2(d_2, d_1) < R_2(d_R, d_R)$ , contradicting (25).

(b) Suppose  $d_2 \geq \tilde{d}_1(d_2)$ . In this case, a change in firm 1's debt level from  $d_1$  to  $d_2$  will not have an effect of firm 1's behavior in the competition stage. Hence  $R_1(d_1, d_2) = R_1(d_2, d_2)$ . Using the definition of  $d_R$ , we have  $R_1(d_2, d_2) \leq R_1(d_R, d_R)$ , which implies  $R_1(d_1, d_2) \leq R_1(d_R, d_R)$ , contradicting (24).

4. *There is no such  $(d_1, d_2)$  with  $d_1 < d_R$  and  $d_2 < d_R$ .* — Suppose  $d_1 < d_2 < d_R$ . Note that we necessarily have  $d_R \leq \tilde{d}(d_R)$ . From Lemma 4,  $R_1(d_1, d_2) < R_1(d_1, d_R)$ . From Lemma 5,  $R_1(d_1, d_R) < R_1(d_R, d_R)$ , so  $R_1(d_1, d_2) < R_1(d_R, d_R)$ , which implies  $R_1(d_1, d_2) < b_R$ , contradicting (24). With the same arguments, the case  $d_2 < d_1 < d_R$  contradicts (25).

5. *There is no such  $(d_1, d_2)$  with either  $d_1 < d_R < d_2$  or  $d_2 < d_R < d_1$ .* — Consider the first possibility. From Lemma 4,  $R_1(d_1, d_2) \leq R_1(d_1, d_R)$ . From Lemma 5,  $R_1(d_1, d_R) < R_1(d_R, d_R)$ . Hence  $R_1(d_1, d_2) < R_1(d_R, d_R)$ , contradicting (24). With the same arguments, the case  $d_2 < d_R < d_1$  contradicts (25).

**Proof of Theorem 2** If both firms' debt equals  $d_R$ , the net profits to a firm  $\Pi(a^*, a^*, d_R)$  will be strictly positive if and only if  $\hat{\omega}(d_R, d_R) > \underline{\omega}$ . From Assumption 2 and Lemma 3, we have that  $\pi^*(d, d, \omega)$  is strictly decreasing in  $d$  for all  $\omega$ . Thus  $\hat{\omega}(d, d)$  is strictly decreasing in  $d$ , and to have  $\hat{\omega}(d_R, d_R) > \underline{\omega}$ , we need  $d_R < \tilde{d}$ . We will show that this is indeed the

case. If  $d_1 = d_2 = d$ , the debtholders lending to firm  $i$  have expected revenue

$$R(d, d) = \Pr(\omega \leq \hat{\omega}(d, d))d + \int_{\hat{\omega}(d, d)}^{\bar{\omega}} \pi^*(d, d, \omega) f(\omega) d\omega.$$

Using Leibniz's rule and dropping the arguments of  $\hat{\omega}$ , we have

$$\begin{aligned} \frac{dR(d, d)}{dd} &= \frac{\partial \Pr(\omega \leq \hat{\omega})}{\partial d} d + \Pr(\omega \leq \hat{\omega}) - \pi^*(d, d, \hat{\omega}) f(\hat{\omega}) \frac{\partial \hat{\omega}}{\partial d} \\ &\quad + \int_{\hat{\omega}}^{\bar{\omega}} \frac{d\pi^*(d, d, \omega)}{dd} f(\omega) d\omega. \end{aligned}$$

Since  $\pi^*(d, d, \hat{\omega}) = d$  and  $f(\omega) \equiv \partial \Pr(\omega \leq \hat{\omega}) / \partial \omega$ , the first and third terms cancel out, so we have

$$\frac{dR(d, d)}{dd} = \Pr(\omega \leq \hat{\omega}) + \int_{\hat{\omega}}^{\bar{\omega}} \frac{d\pi^*(d, d, \omega)}{dd} f(\omega) d\omega. \quad (26)$$

Consider the case in which  $d = 0$ . Then,  $\hat{\omega}(d, d) = \bar{\omega}$ , so we have

$$\left. \frac{dR(d, d)}{dd} \right|_{d=0} = \Pr(\omega \leq \bar{\omega}) = 1. \quad (27)$$

Consider the case in which  $d = \tilde{d}$ . Then,  $\hat{\omega}(d, d) = \underline{\omega}$ , so we have

$$\left. \frac{dR(d, d)}{dd} \right|_{d=\tilde{d}} = \int_{\underline{\omega}}^{\bar{\omega}} \frac{d\pi^*(d, d, \omega)}{dd} f(\omega) d\omega = \frac{dE_{\omega}(\pi^*(d, d))}{dd} < 0, \quad (28)$$

where the inequality follows from Lemma 3. Combining (27) and (28), by continuity there must be at least one  $\check{d} \in (0, \tilde{d})$  where  $dR(d, d)/dd$  evaluated in  $d = \check{d}$  equals zero, and moreover  $d^2R(d, d)/dd^2$  evaluated in  $d = \check{d}$  is negative. Combined with (28), this implies that indeed  $d_R < \tilde{d}$ , which establishes the result.

**Proof of Theorem 3** Consider some other symmetric candidate equilibrium in which every bidder bids some  $b \neq b_R$ . First, consider the case  $b > b_R$ . With  $b_R = \max_d R(d, d)$ , this implies that there is no symmetric equilibrium of the debt stage in which both firms can find financing. Second, consider the case  $b < b_R$ . Consider one of the bidders at the auction, say firm 1. Such a firm can make sure to be among the winners of the auction by bidding  $b_R$ . The debt market is willing to finance the two winning bids  $b$  and  $b_R$ , since

$R(d_R, d_R) = b_R > b$ . Therefore, consider as a candidate equilibrium for the debt stage  $(d_1, d_2) = (d_R, d_R)$ . From Theorem 2, firm 1 then makes positive profits equal to  $\Pi(d_R, d_R)$ . Yet, we may have that firm 2 prefers to defect from this candidate equilibrium of the debt stage, setting a different  $d_2$ . It cannot choose  $d_2 > d_R$ : using Lemma 4 and the fact that  $d_R < \tilde{d}$  (see proof of the previous theorem), we would then have  $R_1(d_R, d_2) < R_1(d_R, d_R)$ , so firm 1 cannot find financing. If firm 2 would prefer some  $d_2 < d_R$ , then that would relax firm 1's financing constraint, while it would increase firm 1's profits, using (21). Thus, by setting  $d_1 = d_R$ , firm 1 can secure profits of at least  $\Pi(d_R, d_R) > 0$ . Hence, firm 1 can indeed make positive profits by submitting a higher bid, which establishes uniqueness at the auction stage. If the maximizer of  $R(d, d)$  is unique, we immediately have uniqueness of the equilibrium. If it is not, we still have that the equilibrium to the auction stage is unique, but that each of the maximizers of  $R(d, d)$  is an equilibrium at the debt stage.

**Proof of Corollary 2** In our model, equilibrium license fees are

$$\begin{aligned} b_R &= \max_d \left\{ \Pr(\omega \leq \hat{\omega}(d, d))d + \int_{\hat{\omega}(d, d)}^{\bar{\omega}} \pi_i^*(d, d, \omega) f(\omega) d\omega \right\} \\ &= \int_{\underline{\omega}}^{\hat{\omega}(d_R, d_R)} d_R f(\omega) d\omega + \int_{\hat{\omega}(d_R, d_R)}^{\bar{\omega}} \pi_i^*(d_R, d_R, \omega) f(\omega) d\omega. \end{aligned}$$

With internal funds, since all firms are identical and information is symmetric, profits will be competed away at the auction stage. The license fees paid in equilibrium are

$$b^{int} = \int_{\underline{\omega}}^{\bar{\omega}} \pi^*(0, 0, \omega) f(\omega) d\omega.$$

We now have

$$b_R < E_\omega(\pi^*(d_R, d_R)) < E_\omega(\pi^*(0, 0)) = b^{int}.$$

The first inequality follows from the definitions of  $b_R$  and the fact that  $d_R < \pi^*(d_R, d_R, \omega)$  for  $\omega < \hat{\omega}(d_R, d_R)$ . The second inequality follows from Lemma 3. This establishes the first statement in the corollary. For the second statement, note that with internal funds,  $d = 0 < d_R$ . The result then follows from proposition 1.



**Proof of Lemma 10** Also in this case,  $dR(d, d)/dd$  is given by (26), i.e.

$$\frac{dR(d, d)}{dd} = \Pr(\omega \leq \hat{\omega}) + \int_{\hat{\omega}}^{\bar{\omega}} \frac{d\pi^*(d, d, \omega)}{dd} f(\omega) d\omega.$$

Obviously  $\Pr(\omega \leq \hat{\omega}) \geq 0$ . From Assumption 2 and Lemma 8, we have  $d\pi^*(d, d, \omega)/dd > 0$ , which establishes the result.

**Proof of Theorem 4** To prove that this is an equilibrium, we need that the three conditions in definition 1 are satisfied. Equilibrium at the competition stage holds trivially. For the debt stage, we need to show that  $(d_R, d_R)$  is an equilibrium, given that both firms have submitted a bid  $b_R$  in the auction stage. For the auction stage, we need to show that no firm is willing to submit a higher bid and that firms submitting  $b_R$  make nonnegative profits.

Consider the *debt stage*. Equilibrium debts  $(d_1, d_2)$  have to satisfy

$$R_1(d_1, d_2) \geq b_R, \tag{29}$$

and

$$R_2(d_2, d_1) \geq b_R. \tag{30}$$

Consider firm 1's best reply to  $d_2 = d_R$ . With  $d_1 < d_R$ , we have from Lemma 9 that  $R_2(d_2, d_1) < R_2(d_R, d_R)$ , contradicting (30). With  $d_1 > d_R$ , the fact that  $d_R = \tilde{d}$  implies that  $R_1(d_1, d_2) = R_1(d_R, d_R)$ . Hence  $d_R$  is a weak best reply to  $d_R$ , which implies that  $(d_R, d_R)$  is an equilibrium for the debt stage.

Now consider the *auction stage*. First note that in the candidate equilibrium considered in the theorem, trivially, a firm cannot make negative profits; firms will be able to find financing, and limited liability in the competition stage precludes negative profits. Hence, a firm cannot strictly improve by submitting a lower bid and losing the auction. Consider a firm that defects by submitting a bid  $b > b_R$ , so we have  $b_1 > b_R$  and  $b_2 = b_R$ . The conditions (6) imply that to be able to have financing for both firms in the new situation,

we need to find a  $(d_1, d_2)$  such that

$$R_1(d_1, d_2) \geq b_1 > b_R, \quad (31)$$

and

$$R_2(d_2, d_1) \geq b_R. \quad (32)$$

The argument proceeds in the following steps:

1. *There is no such  $(d_1, d_2)$  with  $d_1 = d_2$ .* With  $d_1 = d_2$ , we immediately have, from  $d_R = \arg \max R(d, d)$ , that  $R_1(d_1, d_2) \leq R_1(d_R, d_R)$ , contradicting (31).
2. *There is no such  $(d_1, d_2)$  with  $d_2 < d_1$ .* In this case, we have  $R_1(d_1, d_2) \leq R_1(d_1, d_1) \leq R_1(d_R, d_R)$ , with the first inequality following from Lemma 9, and the second from the definition of  $d_R$ . This contradicts (31).
3. *There is no such  $(d_1, d_2)$  with  $d_1 < d_2$ .* Consider two subcases:
  - (a)  $d_1 < \tilde{d}_1(d_2)$ . In that case  $R_2(d_2, d_1) < R_2(d_2, d_2) \leq R_2(d_R, d_R)$ , with the first inequality following from Lemma 9, and the second from the definition of  $d_R$ . This contradicts (32).
  - (b)  $d_1 \geq \tilde{d}_1(d_2)$ . In that case  $R_1(d_1, d_2) = R_1(d_2, d_2) \leq R_1(d_R, d_R)$ , where the equality follows from Lemma 9, and the inequality from the definition of  $d_R$ . This contradicts (31).

**Proof of Theorem 5** Since  $d_R = \tilde{d}$ , we have  $\hat{\omega}(d_R, d_R) = \underline{\omega}$ . Thus, firms never make strictly positive profits, regardless of the realization of  $\omega$ . With limited liability, that implies that their expected profits are zero.

**Proof of Theorem 6** Consider some other symmetric candidate equilibrium in which every bidder bids some  $b \neq b_R$ . First, consider the case  $b > b_R$ . With  $b_R = \max_d R(d, d)$ ,

this implies that there is no symmetric equilibrium of the debt stage in which both firms can find financing. Second, consider the case  $b < b_R$ . Consider one of the other bidders at the auction, say firm 1. Such a firm can make sure to be among the winners of the auction by bidding  $b + \varepsilon$ . Denote as  $\hat{d}$  the debt level for which  $R(\hat{d}, \hat{d}) = b + \varepsilon$ . Thus, both firms can find financing with  $d = \hat{d}$ . With  $R(\hat{d}, \hat{d}) < b_R$ , we have from Lemma 10 that  $\hat{d} < d_R$ , which implies that, at this financing arrangement, both winning firms make strictly positive profits. Yet, we may have that firm 2 prefers to defect from this candidate equilibrium of the debt stage, setting a different  $d_2$ . It cannot choose  $d_2 > \hat{d}$ : using Lemma 9, that would imply that firm 1 can no longer find financing. It may prefer some  $d_2 < \hat{d}$ . In that case, firm 1's profits would even be higher, using 7. This implies that a case in which all firms bid the same  $b < b_R$  is not a Nash equilibrium.

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